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The Conventional, the Theory of Constraints, and the Linear Programming: Three Approaches to the Optimum Production Mix: A Comparative Study

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ABSTRACT

This paper contains research work about three approaches to the optimum production mix problem (OPM); Conventional, theory of constraints (TOC), and Linear programming (LP). The research has two aims, first to develop a solid and clear heuristic describes the conventional and the TOC steps towards OPM. The second is to examine the common belief about the superiority of TOC over the conventional approach.

This study is based on a comparative methodology where the three approaches were applied to a numerical case study and the obtained solutions were compared in terms of which provide the OPM in the form of the highest contribution margin.

As a searching tool for OPM, The Conventional approach is equivalent to TOC and LP in case of a single-constraint system and superior to TOC in situations of single-constraint system with one local constraint.

This paper provided complete and clear heuristics for the conventional and TOC approaches o OPM. And demonstrated that there are situations where the conventional approach is more efficient than the TOC, unlike what is commonly found in the literature.

Keywords: *Optimum Production Mix Problem (OPM), Theory of Constraints (TOC), Linear Programming (LP).*

1. INTRODUCTION

Most of production systems are characterized by their ability to produce and sell different types of products at the same period of time, in its endeavours to attain the strategic goal of the organization as an economical entity, the management of such systems confronted with essential decisions which is how to maximize the economic returns of that system. One aspect in this context is about identifying the combination of the product types and quantities that maximizs the profit. This topic is usually known as the”Optimum Production Mix” (OPM). The issue can be viewed as a method to manage the scarce resource in the organization that works under certain setting and conditions, it has developed as a result of the efforts aimed at maximizing the overall efficiency , and address the imbalance between the supply and demand within the system (organization, firm, factory...etc).

The topic receives since several decades ago, much attention and care from the planners and decision makers, as well as the researchers in several academic fields, including Scientific Management/ Operations Research (SM/OR), engineering & industrial management, and Managerial Accounting.. The most common three approaches in this context are: the Conventional (Accounting) approach, the theory of constraints (TOC) approach, and the linear programming (LP) approach. While the three share the same goal of helping decision makers to identify the combination of the products that maximize the profitability of the system in hand, each one adopts different methodology.

1.1 Literature Review

Several researches have been conducted about the applicability and effectiveness of the theory of constraints (TOC) and linear programming (LP) into defending the optimal production mix. Most of the researches aimed at comparing the two methodologies. The papers by Mabin and Gibson (1998), Kee and Schmidt (1998), Hsu and Chung (1998) , Verma (1997),Bhattacharya and Vasant (2007), Blakrishna and Cheng (2000), Luebbe and Finch (2001) Lea and Fredendall (2002), Boyd and Cox (2002), Mabin and Davies (2003), are examples. De Souza and Manfrinato (2007).

The Conventional method received less interest (Goldratt, 1983, Sheu et al, 2003) considered the conventional approach as an outmoded way to deal with the OPM.

Ray E., Sarhar B., and Sanyal S. K.(2008) propose a methodology based on using Laplace criterion under TOC and compared it with the standard cost accounting and standard TOC approaches in finding the OPM under outsourcing case. Blakrishna and Cheng (2000) studied the relationshipbetween TOC and linear programming .Tanhaei and Nahavandi (2013) show that TOC is unable to find the optimum product mix in the case of more-than-one constraint in the system and propose a relatively simple algorithm to deal with the situation. Mansuradad, Daneshi, and pirzad (2013) studied a special case through which they proof that the conventional and TOC approaches may miss the OPM.

1.2 Motivation

The primary focus in the above mentioned papers was evaluating the performance of TOC and LP when dealing with cases involving more than one bottleneck. The findings suggest that TOC does not offer the optimum solution under these circumstances.

This research broadened the searching frame to include the conventional approach besides TOC and LP, furthermore, and unlike the previous research that focused on the existence of two constraints simultaneously, this study addresses a special case where there is scarcity in one resource belonging to a specific product along with one constraint. To the knowledge of the researcher, this case has not been dealt with in any previous research.

In the context of this paper, and to distinguish the two situations, the term Constraint or Bottleneck will be used to denote the most scarce resource used by all competing products, on the other hand, the term local constraint (LC) will be used to refer to the scarce resource which relates to one product only and is not subject of competition between all of the products.

1.3 Methodology

The main objective for the paper is to compare the efficiency of the Conventional and the TOC approaches as an OPM tool under the situation described above. The current study will provide two heuristics, each of which describes the sequential steps to determine the OPM according to the Conventional or TOC approaches, which is incidentally the missing element in all of the reviewed papers.

To test the validity of the proposed heuristics, each one will be applied to a simple case study characterized by a single constraint. LP is an approach that considered by many researchers as the most effective OPM approach, hence it will be employed in this study to play the role of the reference, where the results to be obtained from the two proposed heuristics will be compared with the results provided by the LP. The conformity in the solutions will be taken as proof of the validity of the proposed heuristics.

In next stage, the case study will be modified a bit to reflect the special situation described above, then the whole analysis will be redone in order to obtain the solutions for the modified case, hence, the comparison of the results will be conducted and the results of the study to be extract.

1.4 The Optimum Production Mix (OPM)

There are various definitions of product mix, Patterson (1992) defined it as a problem consisting of both the identification and the quantification of each product to be produced. The objective is to maximize profit (or minimize loss) for the organization. A more suitable definition for this discussion describes the product mix decision as the quantity of each type of product, which are competing for the available resources, to be produced and sold in a given period, that guarantee maximization of the firm’s economic results (Fredendall & Lea, 1997).

One strategy, among many others, that might serve the purpose, is maximizing the returns on the resources being used, that can be done through directing the available resources to serve the activities that contribute to producing and selling of the most profitable products. In this sense, the management has to allocate the resources in a way that maximizes the ratio between inputs and output of the whole system, in other words maximizing overall efficiency of the entire system.

2. CASE STUDY

A specific company manufactures and sells three types of products, X1, X2 and X3. The manufacturing facility consists of four processing stations S1, S2, S3, and S4. Selling prices per unit are 180 for product X1, 160 for product X2, and 250 for product X3 (in monitory units). The layout process is depicted in Figure (1) below, the numbers inside the boxes indicate the times in minutes needed to process each product in that station. Each of the four stations is available for 9,000 minutes/month, and all the company’s costs and expenses - apart from those related to the raw materials and direct labour is 8,000/month.

To make one unit of any of the three products, (M1, M2, M3, and M4) is to be used. Table (1) below shows the costs of the materials needed, and the direct labour times and costs, as well as the machining times required for producing each of the three products.

Table (2) shows the amounts of the resources available to support the production plan of the forthcoming month.

The question is how many of products X1, X2, and X3 must be produced and sold in order to maximize the total profit of the company during the next month. If the expected demand for the three products X1, X2, and X3 during the month is 200,100 and 180 units respectively?

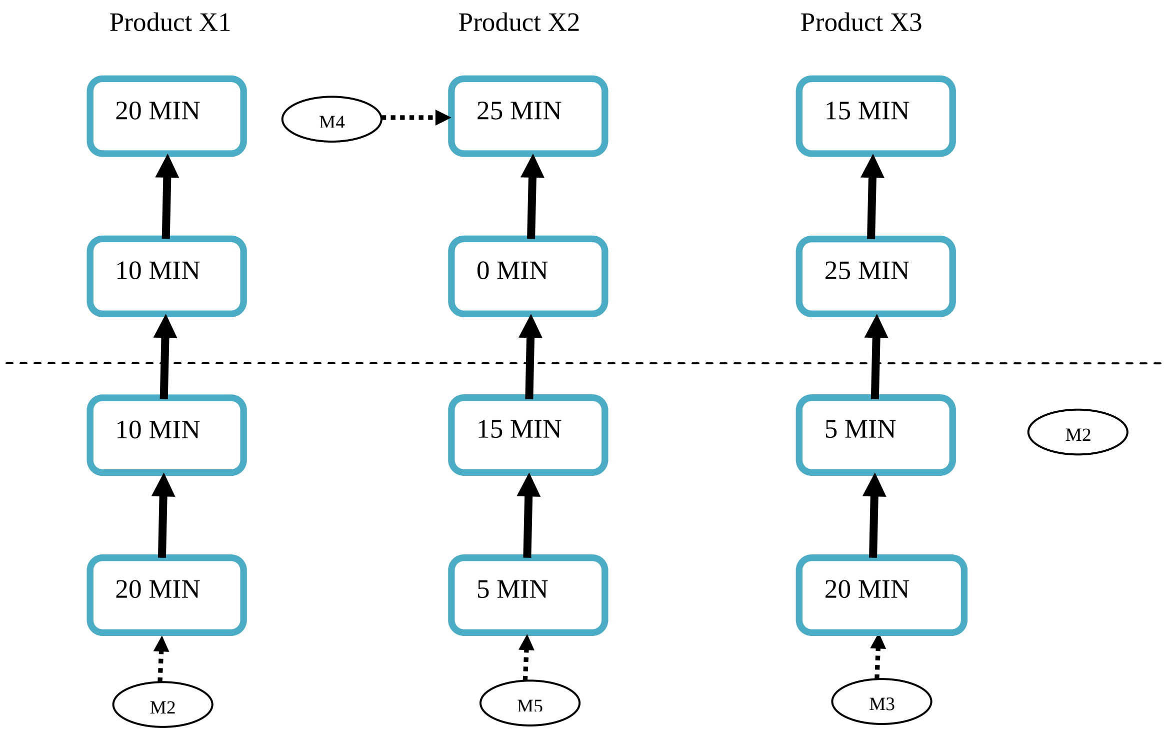

Fig. 1. The Process Layout

Table 1: Variable Costs & Processing Times

	X1	X2	X3
Variable Costs/Unit			
Direct materials $/unit	90	45	140
Direct Labor $/unit	10	8	14
Processing times			
Direct Labor Min/Unit	40	50	80
Machinery Min/Unit	60	45	65

Table 2: Resources Available

	X1	X2	X3
Materials (in $)	20,000	5,000	30,000
Direct Labor (in Minutes)	14,000	6,000	15,000
Machinery in Minutes	9,000 For Every Station		

3. THE CONVENTIONAL APPROACH TO OPM

The conventional approach is built around the contribution margin concept (CM), the well-established measure in the managerial accounting, and the widespread in practice.[1]
In the context of OPM, there are two meanings of the term; the first refers to the difference between the selling price for one unit of product (Xi) and its variable production cost.[2]

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[1] *It should be noted here that the difference between the profit per unit and Contribution Margin of the same is the fixed costs, which is- by definition- a constant amount within the relevant range of production, regardless what and how much the system produces, therefore it is not a relevant variable into the OPM issue.*
[2] *This includes direct materials, direct labor, and direct variable overhead costs.*

This can be denoted as:

$$CM(Xi) = S(Xi) - DM(Xi) - DL(Xi) - VOH(Xi)$$

Where:

(i)is the product index

CM(Xi)is the Contibution Margin of product i.

S(Xi)is the selling price for product i.

DM(Xi)is the direct material neededto pruduce one unit of product i.

DL(Xi)is the direct labur neededto pruduce one unit of product i.

VOH(Xi) is the variable overhead cost for one unit of the pdoduct i

The second meaning of the term in this regard is the total contribution margin (TCM) which is the sum of the contribution margin for each product multiplied by the number of units sold (or to be sold) of that product.

$$TCM = \sum_{i=1}^{n} CM(Xi)$$

On the basis of the previously given definition to OPM as the products' mix that realizes the highest Total Contribution Margin. It can be concluded that maximization of the previous function leads to the desired combination of products.

$$\text{MAXIMIZE: } TCM = \sum_{i=1}^{n} CM\ (Xi)$$

The "maximization process" here is of two dimensions, the first is about identifying the product (Xi) among the list of the products that can be produced and sold , and the second is determining the quantity of that product to be produced and sold. It must be noted that the maximization in the previous equation does not mean striving towards increasing the contribution margin of each particular product to the highest possible level, but it means maximization of the total contribution margin generated by the whole system. The difference between the two meanings is more fundamental than it might appear, that is while the first is related to optimality at the product level, the second is connected to the optimality at the system's level (globle), the direct product of this segregation draws the line and differentiates operational from strategic management and establishes the general principle which affirms that the sum of local optima is not equal to the optimum of the whole.

Stating the two concepts are different and not refer to the same thing, gives rise to the possibility that the two might be in conflict one way or another. Whereas, the realization of one will be on the account of the other i.e. realizing higher level in the local optima leads to a decline in the global optima , the ultimate and actual goal for the management.

It is now well-established in the literature of Industrial Management that enhancing the performance of a system on the bases of local optimization does not result in increased performance for the whole manufacturing plant, and the local optima approach leads to a sum of local optima which is often below the overall optimum performance.[1] Therefore, the optimal production mix must be obtained by looking at all constraints of the system all together, (Slack et al, 2001).

3.1 The Conventional Heuristic Approach for OPM:

According to this approach, determining the OPM can be achieved through the following steps:

1- Calculate the (CM) for the product (Xi) for i = (1, 2,….n), and rank them discerningly.
2- Let i = 1
3- Determine the available resources in relation to the product (Xi)
4- Calculate the number of units of product (Xi) that can be produced.
5- If the outcome of step 4 is = 0, then stop.
6- Compare the figures obtained in step 4 with product (Xi) demand and select the smallest.
7- Direct the resources to produce the number of units of (Xi) determined in the previous point.
8- Advance (i) by one.
9- If i > n then stop.
10- Go to step 3**.**

3.2 Theory of Constraints' Approach to OPM

The theory of constraints is based on the assumption that every system, such as a manufacturing workshop contains at least one constraint at the time that hinders the system from realizing higher profits. Without the latter, the system could possibly accumulate unlimited profits. The theory received wide attention from practitioners and academic researchers since the publication of Goldratt's management novel "The Goal" in 1984. (see Goldratt and Jeff Cox 2004)

According to TOC, the goal of all companies – as economical entities – is to generate "money". The theory proposed the term "throughput" as a specific measure in this regard, in TOC terminology, throughput (T) of one unit of product (Xi) in a manufacturing system is defined as the revenue generated by the system through the production and sale of that product minus the total variable cost (TVC) which is limited to the direct materials that go into that particular product.

Therefore: T (Xi) = P (Xi) – DM (Xi),

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[1] Local Optimum is the conventional approach which assumes that the overall optima value of the objective can be obtained by ensuring local optima, a number of practical approaches have been proposed which help operations managers avoid falling into the local optima trap. Such as focused factory approach and kanban-based production systems that can be characterized as a method which attempts to achieve better performance by avoiding local optima.

Where P(*Xi)* is the selling price and DM (*Xi*) is the total cost of the direct materials,for the product *(Xi).*

And the throughout for the whole system (TT)

$$= \sum_{i=1}^{n}(PXi - DM\ Xi * Qi).$$

Where (Qi) is the quantity of product (xi) sold during the period.

In its strive to maximize the total throughput of the whole system, TOC focuses on the management of the constraint(s) of the system; hence, the priority is to be given to products having more throughput and consumes less of the constraints, Roodposhti (2007), compared to the throughput it creates. Thus, the amount of throughput per unit of constraint is the criteria for product manufacturing prioritization under TOC. (Lockamy 2003).

While the traditional definition of productivity focuses on "output per unit of time", TOC emphasizes "sold product", rather than simply "output", because unsold product does not generate revenues (Sheu et al, 2003) which is the targeted element for maximization according to TOC.

3.3 A Proposed Heuristic for TOC's Approach to OPM

In order to select the product mix through TOC approach, and building on the general content of points in section above, the following steps are to be followed:

1- Calculate the (MC) for the whole products' range.
2- Calculate the required resources to meet the demand for the whole products range.
3- Compare the required resources to meet the demand with the available ones for each of the products i.e. (Xi) for i = 1, 2,…, n.
4- Identify the bottleneck (BN), the limiting factor that hinders the system from meeting demand in full.
5- Calculate the CM /BN ratio for each of the products and rank the products descendingly.
6- Let i =1.
7- Find out the highest possible quantity of product (Xi) that the limiting factor can support.
8- If the outcome of step 7 is less than 1, then stop.
9- Compare the figure obtained in step 7 with the demand for (Xi) and select the smallest.
10- Direct the resources to produce the number of units of product (Xi) determined in the previous step.
11- Advance (i) by one.
12- If I ≥ n, then stop.
13- Go to step 7.

3.4 Linear Programming Approach

Linear programming is an optimization method for finding the optimum solution in the form of maximum or minimum values of an objective under some constraints, where the objective and constraints are expressed in linear form. It is a branch of the mathematical programming technique. Other versions include integer programming (IP), that is used in problems requiring integer solutions; nonlinear programming (NLP), where the objective and/or one constraint or more is nonlinear functions; and goal programming (GP), for problems with multiple objectives.

The analyses in this paper are limited to linear programming, under which, there is a number of conditions that must be fulfilled in order to consider the LP as an applicable tool for the case in hand, these conditions are presented in the form of four assumptions underlie this methodology, that need to be held true in the system under investigation.

Due to the importance of these hypotheses, and to the need to ensure its fulfilment when adopting this approach, it is going to be discussed briefly in the coming paragraph.[1]

Assumption1. Proportionality: Proportionality means that each decision variable (the number of units (n) of the product in the objective and constraint functions must appear with a constant coefficient. Proportionality implies also that the revenue and cost functions of the product (Xi) are assumed to be fixed throughout the entire range of activity levels (the relevant range), hence, the marginal rate of contribution of each variable to the objective remains constant, that is each additional unit of the product (Xi) produced by the system consumes the same quantity of resources, and yields the same contribution margin as the previous unit.

In real world, this may not always hold true. Economies of scale, for instance, reflect variations in costs and profit margins as production levels change. Price discounting for certain preferred customers or region also violates the proportionality assumption. But, if the variable coefficient is representative of the average contribution margin rate for that product, the assumption can be said to reasonably hold.

Assumption 2. Additively means that variables are added to or/and subtracted from but never multiplied or divided by each other.The function MAXIMIZE: 80A*107B +106C is not acceptable.In the objective function, additivity implies that the contribution of the variables to the objective is assumed to be the sum of their individual weighted contributions. In the case of the OPM, total CM is the sum of the individual product's CMs. In the constraints side, additivity implies that total resource usage is likewise the sum of individual resource usage per product.

Proportionality and additivity together compose the linearity. The broader implication of linearity is that the variables are assumed to be mutually independent. In other words, the products (X1, X2..., Xn) are assumed to be

[1] *This section draws heavily on the contents of the following site: http://agecon2.tamu.edu/people/faculty/mccarl-bruce/mccspr/new02.pdf. Visited in Feb. 2016.*

neither complements nor substitutes of each other, hence, there is no interaction between the products...If linearity cannot be hold; the problem would call for a nonlinear programming solution approach.

Assumption 3, Divisibility: Divisibility means that the variables (number of products X1, X2,..., Xn) can take fractional values. This follows from the fact that objective function is linear continuous, and the coordinates of the constituent points need not always be integers. If production system is conceived of as a continuous process, divisibility is usually not an obstacle. Where fractional parts of the complete units can often be interpreted as work in process (WIP) that can be finished on the next production period. At any rate, if integer solutions are required, the analyses can be done using Integer Programming.

Assumption 4. Certainty. means that the problem is assumed to have no probabilistic elements whatsoever,this is technically never true in the real world; some degree of uncertainty is always present. However, for short-term planning, the level of uncertainty tends to be minimal and one can often work under the assumption of complete certainty and then take small parameter variables into account with sensitivity analysis.

3.5 The Mathematical Formation of the Original Case Study

Maximize the following objective function: 80X1 + 107X2 + 96X3 Subject to the following constraints:

$90X1 \leq 20000$ (Capacity Constraints for direct materials for product X1).

$45X2 \leq 5000$ (Capacity Constraints for direct materials for product X2).

$140X3 \leq 28000$ (Capacity Constraints for direct materials for product X3).

$40X1 \leq 14000$ (Capacity Constraints for direct Labour for product X1).

$50X2 \leq 6000$ (Capacity Constraints for direct Labour for product X2).

$80X3 \leq 15000$ (Capacity Constraints for direct Labour for product X3).

$20X1+5X2+20X3 \leq 9000$ (Capacity Constraints for process times in station 1).

$10X1+15X2+5X3 \leq 9000$ (Capacity Constraints for process times in station 2).

$10X1+0X2+25X3 \leq 9000$ (Capacity Constraints for process times in station 3).

$20X1+25X2+15X3 \leq 9000$ (Capacity Constraints for process times in station 4).

$X1 \leq 200, X2 \leq 100, X3 \leq 180$ (Demand constraints).

$X1, X2, X3 \geq 0$ (Non-Negativity condition).

4. IDENTIFING THE PRODUCT MIX UNDER THE THREE APPROACHES

The case study provided in section 2 is going to be used to calculate the OPM under each of the three approaches. Since the fixed cost remains constant regardless what and how much the system would produce and sell, and to avoid going into discussing the different accounting theories in allocating the fixed costs to the products, OPM is going to be determined under the conventional and TOC approaches on the basis of the Contribution Margin (CM). Where the product mix that gives maximum contribution is to be considered the best in terms of profitability.

The conventional and the TOC approaches depend on ordering the products in a priority list. But each one of them has its own criteria in ordering the products in that list.

Under the conventional approach the higher priority goes to the product with the higher (CM) per unit. Appendix (1) exhibits the implementation of the presented heuristic in section 3-2 above. Tables 1/A/2, 1/A/3 and 1/A/4 depict three runs of the loop described in steps from 3 to 10 in the proposed heuristic. The result of the calculations suggests the production of 190, 100, 180 units of products X1, X2, and X3 respectively, that mix yields the maximum profit this company could generate under the conditions illustrated in the case.

Under the TOC approach the top priority goes to the product with the highest ratio between the CM per unit and the requirements of that unit in the overloaded resource (the constraint).

From Table1/B/2, it is obvious that there's only one constraint which is station S4, requiring 9,200 minutes where only 9000 minutes are available.

Calculations based on the heuristic stated in section 4-4 suggest producing of 190, 100, 180 units of products X1, X2, andX3 respectively. (Tables 1/B/3, 1/B/4, 1/B/5)

LP approach does not depend on ordering the products in a priority list, in contrary, it deals with all the products ,requirements and restrictions as one package, where the objective and the constraints are formulated in mathematical models that take the form of first order equations or inequalities, and is resolved simultaneously.Calculations suggest producing 190, 100, 180 units of products X1, X2, andX3 respectively.

the above shows that the three approaches offer the same solution to the case in hand[1], this firstly confirms the validity of the proposed heuristics, and secondly it supports the view that the three approaches are equal in terms of their efficiency in finding the best solution for the production mix in similar cases.

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[1] *This genrates an amount of (27,080) as aprofits.*

5. THE MODIFIED CASE STUDY

Up to this point, the analysis showed that the three approaches offer the same solution, this raises the following question; on the bases of the previous results, to what extent one can safely infer that the three approaches are equally appropriate as tools for the OPM?

To answer this question, further investigations to test the performance of the three approaches under various conditions are needed. For the sake of simplicity, such investigations will be conducted using the same case study, with limited modification.

The case study includes three solution variables (products), and four groups of constraints, namely, demand, selling price, variable costs, and the resources available. This paper does not intend to investigate the effect that a change in each parameter may have on the solution variables, whereas such investigation can be performed more effectively by means of a simulation, which is beyond the scope of this study. Instead, the paper will assume a change in one variables only, that is the cost of the direct materials for product X3 (Hereinafter DMX3), so that it became 185/unit instead of 140/unit. This will make the available direct materials for product X3 less than what it is required to meet the expected demand, and that would create an additional but special bottleneck in the system besides the current constraint S4. It must be noted here the difference between the cases we are dealing with here and the similar case where there are more than one constraint within the system at the same time, (i.e. multi-constraint system). The situation depicted herein is related to the existence of a shortage in a single resource linked to one product, not to all of the products, while the multi-constraint-system situation means the existence of shortages in more than one resource each of which is linked to all products, in other words, we are here dealing with the case of one constraint plus a local constraint, but not with the case of two constraints.

In light of this understanding, the analysis was performed again in order to find OPM under the three approaches, following the same steps described previously and using the modified case. This has resulted into the outcomes shown in the following table.[1]

[1] *Reader who wants to examine the calculations is advised to consult the Tables in appendix 2.*

The Three Approaches': Solutions to the Modified Case Study

Product	Conventional	TOC	LP
X1	200	200	200
X2	100	100	100
X3	151.35	166.6[2]	151.35

6. SUMMARY

Three approaches were used to determine the optimal product mix, OPM; the analyses were based on a hypothetical case study of two versions; original and modified one.

Analyses based on the original case showed that the three approaches offer the same solution.

The modified version of the case study was developed by changing one resource, which is the direct materials for product X3 (DMX3).

Analysis of the modified case study showed that the three approaches offered different solutions, these are as follows:

I. LP and the Conventional approachs suggested the same production mix ,which yields a contribution margin of 34,418.9.

II. TOC approach suggested a different production mix that yields a 35,196.9 , the solution is inapplicable since it is inconsistent with the value of direct materials needed to produce the proposed 166.6 units of product X3. therfore, the approach failed to provid the correct answer.

7. DISCUSSIONS

Among OPM approaches, the Conventional, theory of constraints and linear programming are the more common approaches ; the goal is always to maximize the efficiency of the firm as a whole, and each represents a unique systemic approach to improve the overall performance.

The Conventional and TOC approaches adopt similar but still different heuristics in order to find OPM. Under the Conventional approach, the product that contributes the most to the firm's contribution margin (CM) is the product to be given the first priority for production. According to TOC, the product that brings the best relationship between throughput (T) and the consumed constraint units CU (T/CU) is the preferred product. On this bases ,each of the

[2] *It must be mentioned here that the accounting system that associates TOC (the throughput accounting), considers only the sold products as production. Thus, partially finished products (Work-In-Progress) are not considered as part of the production values during the period. The 0.6 of product X3 value included here in the calculations just for easing the comparisons.*

two approaches develop a priority list that contain all the products that can be produced and sold during the planning period, put in descending order. The solution offered by the two approaches therefore suggests that the maximum possible amount of the top-list product to be produced, and then using any remaining capacity in the bottleneck to produce the next product in the priority list and so on. A notable point here is that TOC does not consider labour cost as variable cost, and, therefore, it is not included in the contribution margin calculations as is the case with the conventional approach.

The foregoing indicates that any differences between the solutions that might be proposed by the two approaches for the same situation are fundamentally due to the variations into ranking the products in the priority lists developed by each of the two approaches, and this in turn is imputed to the difference in the prioritization criteria that is used under each approach.

Linear programming is considered to be the most efficient tool regarding OPM; therefore, it is used in many studies to validate the optimality of the solution obtained by the other approaches. On the other hand, the conventional and TOC approaches ar heuristic-based approaches,therefore, it is more often used by managers who lack the applied mathematical skills needed for linear programming, especially when it comes to making sure that all the assumptions of the method are fulfilled.

The main criticism addressed to the Conventional approach in this context is that it does not differentiate between constraint and non-constraint resources. Therefore, if a processing station in a manufacturing facility becomes idle, whether it is a bottleneck or non-bottleneck, it is assumed to receive an equal attention from the management. While TOC states that "an hour lost at the bottleneck is an hour lost for the whole plant because it is irretrievable, but an hour lost at a non-bottleneck is only a mirage".

LP, as well, recognizes the difference between bottleneck and non-bottleneck conditions. This is shown by the way it deals with the possible improvement in the objective function, known as the shadow price.[1] In this regard, linear programming assigns a non-zero shadow price to bottleneck constraints, while it assigns zero shadow prices to non-bottleneck constraints.

In the modified case study, the cost of material for product X3 is assumed to increase, from 140 to 185, this alteration results in scarcity in direct materials for that product (DMX3), and created a bottleneck of a special nature, termed a local constraint (In addition to resource S4, which already existed as a global constraint). While resource S4 still carries a 102% load as before, resource (DMX3) now assumes 118% load. Even as the load here is higher than the load in (S4), it practically effects the production of X3 only; hence it lacks the property of being global.

As it can be seen from the headings in Tables (1/A/2) to (1/A/4**)** in appendix 1, (DMX3) is from within the allocated (assigned) resources, which means that it is not subject to competition among all products. If a resource of this type falls short in meeting the demand for Xi, it would be representing a constraint for that product only, but not to any of the other products. Therefore, that resource is a local point of suffocation but not global, and it would not be considered as a bottleneck. Hence, according to TOC approach, it would not be taken into account as such. Therefore, and as a direct result of ignoring this local constraint, the proposed solution would not be applicable.

The other two approaches do not ignore this local constraint, and so they offer an identical and applicable solution. This is what makes the solution proposed by TOC different from the solution provided by the other two approaches.

This point can be illustrated using the chart at the end of Appendix 2, which represents the graphical solution provided by the LP to the modified case study. The feasible solution area in the graph is the rectangle determined by the horizontal line that represents the constraint function of DMX3 (M for X3) and the vertical line represents the constraint function of demand for X2 (D for X2). Because TOC does not consider the first constraint, the rectangle of the feasible area will expand vertically up to the point where its top-right angle become in touch with the line that represents the constraint function of "Time AVA in S3" the new limiting factor. This point corresponds to 166.6 on the vertical axis that represents the variable X3. (Follow the dotted line in the graph), which gives the same solution provided by the conventional and the LP approaches.

Under these conditions, when the previously suggested TOC approach is followed, resource S4 would be regarded as the bottleneck, not for it is being more overloaded than any other resource including (DMX3), but because it is the most over loaded common resource that hinder the system as a whole from producing more products and achieving better performance; so the TOC solution would maintain the same bottleneck (the S4) in the modified case as the it is in the original case study. However, only 151.35 units of X3 can now be produced (not 166.6 units), that is due to the lack of capacity in (DMX3). Under this solution, the resource S4 would be operating at 9000/9000 = 100% load, and (DMX3) would be loaded at 33300/28000 = 118.9 – 100 = 18.9% over its full capacity. Hence, the solution is inapplicable.

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[1] Shadow price of a constrint is the marginal change in the objective function as result of increing the right-hand saide of that constrint by one unit.

8. CONCLUSIONS

Almost all the previous researches are in agreement about the superiority of TOC over the conventional approach as a tool for OPM. Therefore; it is concluded that the returns of any manufacturing system that implements this approach will be lower than the returns of the similar system implementing TOC approach.

This research assures that the Conventional approach is equal to TOC approach when the system contains one bottleneck. Furthermore, it showed that the conventional approach is superior to TOC when there is a single constraint besides one or more local constraints.

9. FURTHER RESEARCH OPPORTUNITIES

The findings of this paper answered one question, but give arise to several others, such as:

- What is the expected performance of the conventional and the TOC heuristics in case of multi-constraints systems and the interactive constraints?
- To what extent the proposed heuristic can be regarded as a form of dynamic programming?

Each of the two questions can be subject to further research efforts, and it would add to the body of knowledge in the subject.

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Appendix 1 (The Original Case Study)
A: Conventional Approach to OPM

Table (1/A/1): Contribution Margin

	X1	X2	X3
Selling Price	180	160	250
Mines variable costs/unit			
Direct materials $/unit	90	45	140
Direct Labor $/unit	10	8	14
Contribution margin/unit	80	107	96
Ranks of the products	**3**	**1**	**2**

Table (1/A/2): Product Ranked the First: (X2)

	Assigned Resources (Materials, Labor)		**Common Resources (Process Stations)**			
	M	**L**	**SI**	**S2**	**S3**	**S4**
Resources available to produce (X2)	5,000	6,000	9,000	9,000	9,000	9,000
Requirements per unit	45	50	5	15	0	25
Units possible to produce	111	120	1,800	600		360
Limiting factor	Demand = 100 unit					
Demand	100					
Units to produce	100					
Requirements	4,500	5,000	500	1,500	0	2,500
Capacities balances	4,500	5,000	8,500	7,500	9,000	6,500

Table (1/A/3): Product Ranked the Second: (X3)

	Assigned Resources (Materials, Labor)		**Common Resources (Process Stations)**			
	M	L	SI	S2	S3	S4
Resources available to produce (X3)	28,000	15,000	8,500	7,500	9,000	6,500
Requirements per unit	140	80	20	5	25	15
Visible units possible to produce	200	188	425	1,500	360	433
Limiting factor	Labor = 188 unit					
Demand	180					
Units to produce	180					
Requirements	25,200	14,400	3,600	900	4,500	2,700
Capacities balances	2,800	600	4,900	6,600	4,500	3,800

Table (1/A/4): Product Ranked Third: (X1)

	Assigned Resources (Materials, Labor)		**Common Resources (Process Stations)**			
	M	L	S1	S2	S3	S4
Resources available to produce (X1)	20,000	14,000	4,900	6,600	4,500	3,800
Requirements per unit	90	40	20	10	10	20
Units possible to produce	222	350	245	660	450	190
Limiting Factor	S4 = 190 Units					
Demand	200					
Units to produce	190					
Requirements	17,100	7,600	3,800	1,900	1,900	3,800
Capacities balances	17,100	7,600	1,100	4,700	2,600	0

Appendix 1/B: TOC Approach to OPM

Table (1/B/1): Contribution Margin

	X1	**X2**	**X3**
Selling price	180	160	250
Mines variable costs/unit			
Direct materials $/unit	90	45	140
Contribution margin/unit	90	115	110
Ranks of the products	**3**	**1**	**2**

Table (1/B/2) Identifying the Bottleneck

Product	**Demand**	**Assigned Resources (Materials, Labor)**			**Common Resource (Process Stations)**				
			M	L		S1	S2	S3	S4
X1	200	AVAILBLE	20000	14000	AVAILBLE	9000	9000	9000	9000
		REQUIRED	18000	8000					
X2	100	AVAILBLE	5000	6000					
		REQUIRED	4500	5000	REQUIRED	8100	4400	6500	9200
X3	180	AVAILBLE	28000	15000					
		REQUIRED	25200	14400					

Table (1/B/3): Products Priorities

Product	Contribution Margin (CM)	Time in the Bottleneck (Minutes in S4)	CM/BN ratio	Priorities*
X1	90	20	4.50	3
X2	115	25	4.60	2
X3	110	15	7.33	1

*****Priority is calculated by dividing the CM of the product by the time it takes on the constraint that governs the output of the system (Bottleneck).

Table (1/B/4):Production Plan

Product	Priorities	Time in the BN per unit	Demand	Units can be Produced	Units to be Produced	Time in the BN per Product
X1	**3**	20	200	190*	190	3,800
X2	**2**	25	100	252**	100	2,500
X3	**1**	15	180	600***	180	2,700

* (9000-2700-2500)/20 = 190, ** (9000-2700)/25 = 252,***9000/15 = 600.

Appendix 1/C: LP Approach to OPM

The Mathematical Formation of the Original Case Study

Maximize the following objective function: 80 X1 + 107 X2 + 96 X3
Subject to the following constraints:
$90 X1 \leq 20000$ (Capacity Constraints for direct materials for product X1).
$45X2 \leq 5000$ (Capacity Constraints for direct materials for product X2).
$140X3 \leq 28000$ (Capacity Constraints for direct materials for product X3).
$40X1 \leq 14000$ (Capacity Constraints for direct Labour for product X1).
$50X2 \leq 6000$ (Capacity Constraints for direct Labour for product X2).
$80X3 \leq 15000$ (Capacity Constraints for direct Labour for product X3).
$20X1 + 5X2 + 20X3 \leq 9000$ (Capacity Constraints for process times in station 1).
$10X1 + 15X2+5X3 \leq 9000$ (Capacity Constraints for process times in station 2).
$10X1 + 0X2 + 25X3 \leq 9000$ (Capacity Constraints for process times in station 3).
$20X1 + 25X2+15X3 \leq 9000$ (Capacity Constraints for process times in station 4).
$X1 \leq 200, X2 \leq 100, X3 \leq 180$ (Demand constraints).

Appendix 2 The Modified Case Study: (Direct Materials for X3 is 185/unit Instead of 140/unit)

A: Conventional Approach to OPM

Table (2/A/1): Contribution Margin

	X1	X2	X3
Selling price	180	160	250
Mines variable costs/unit			
Direct materials $/unit	90	45	**185**
Direct Labor $/unit	10	8	14
Marginal contribution/unit	80	107	51
Ranks of the products	**2**	**1**	**3**

Table (2/A/2): Product Ranked the First: (X2)

	Assigned Resources (Materials, Labor)		Common Resources (Process Stations)			
	M	L	SI	S2	S3	S4
Resources available to produce (X2)	5,000	6,000	9,000	9,000	9,000	9,000
Requirements per unit	45	8	20	5	25	15
Units possible to produce	111	750	450	1,800	360	600
Limiting Factor	M = 111 Unit					
Demand	100					
Units to produce	100					
Requirements	4,500	800	2,000	500	2,500	1,500
Capacities balances	500	5,200	7,000	8,500	6,500	7,500

Table (2/A/3): Product Ranked the Second : (X1)

	Assigned Resources (Materials, Labor)		Common Resources (Process Stations)			
	M	L	SI	S2	S3	S4
Resources available to produce (X1)	20,000	14,000	7,000	8,500	6,500	7,500
Requirements per unit	90	40	20	10	10	20
Units possible to produce	222.222	350	350	850	650	375
Limiting Factor	Demand = 200 Unit					
Demand	200					
Units to produce	200					
Requirements	9,000	4,000	2,000	1,000	1,000	2,000
Capacities balances	11,000	10,000	5,000	7,500	5,500	5,500

Table (2/A/4): Product ranked the THIERD: (X3)

	Assigned Resources (Materials, Labor)		**Common Resources (Process Stations)**			
	M	**L**	**S1**	**S2**	**S3**	**S4**
Resources available to produce (X3)	28000	15000	7000	8500	6500	7500
Requirements per unit	185	80	20	10	10	20
Units possible to produce	151.35	188	350	850	650	375
Limiting Factor	151.35					
Demand	200					
Units to produce	151.35					
Requirements	28,000	12,108	3,027	1,514	1,514	3,027
Capacities Balances	0	2,892	3,973	6,986	4,986	4,473

Table (2/A/5) Income statement

	X1	**X2**	**X3**	**Totals**
Sells	200*180=36,000	100*160=16,000	151.35*250=3,7837.5	89,837.5
Direct Materials	200*90=18,000	100*45=4,500	151.35*185=27,999.75	50,499.8
Labor costs	200*10=2,000	100*8=800	151.35*14=2,118.9	49,18.9
Contribution M.				34,418.9
Fixed costs				8,000
Gross Profit				26,418.9

Appendix 2/B: TOC Approach to OPM

Table (2/B/1): Contribution Margin

	X1	**X2**	**X3**
Selling price	180	160	250
Mines Variable costs/u			
Direct materials $/u	90	45	**185**
Contribution Margin/u	90	115	**65**

Table (2/B/2) Identifying the Bottleneck

Product	**Demand**	**ASSIGNED RESOURCES (Materials ,Labor)**			**COMMON RESOURCES (Process Stations)**				
			M	**L**		**S1**	**S2**	**S3**	**S4**
X1	200	AV	20,000	14,000	AV	9,000	9,000	9000	9,000
		RE	18,000	8,000					
X2	100	AV	5,000	6,000					
		RE	4,500	5,000	RE	8,100	4,400	6,500	**9,200**
X3	180	AV	28,000	15,000					
		RE	**33,300**	14,400					

AV = AVAILABLE RE=REDUIRED

Table (2/B/3) :Products' priority

Product	Contribution Margin (CM)	Time in the BN (minutes in S4)	C M / Minute	Priority
X1	90	20	4.5	**2**
X2	115	25	4.6	**1**
X3	**65**	15	4.3333	**3**

Table (2/B/4) :Production plan

Product	Priority	Time in the BN /unit	Demand	Units can be produced	Units to be in plan	Time in the BN per product
X1	**2**	20	200	360 *	200	4,000
X2	**1**	25	100	325 **	100	2,500
X3	**3**	15	180	166.6 ***	166.6	2,500

*9000/25=360 ,**(9000-2500)/20=325 ,***(9000-4000-2500)/15=166.6

Table (2/B/5) Income statement

	X1	X2	X3	Totals
Sells	200*180=36,000	100*160=16,000	166.6*250=41,650	93,650.00
Direct Materials	200*90=18,000	100*45=4,500	166.6*185=30,821	53,321.00-
throughput				40,329. 00-
Labor costs	200*10=2,000	100*8=800	166.6*14=2,332.4	5,132.40-
CM				34,418.9
Fixed costs				8,000.00-
Total profits				27,196.60

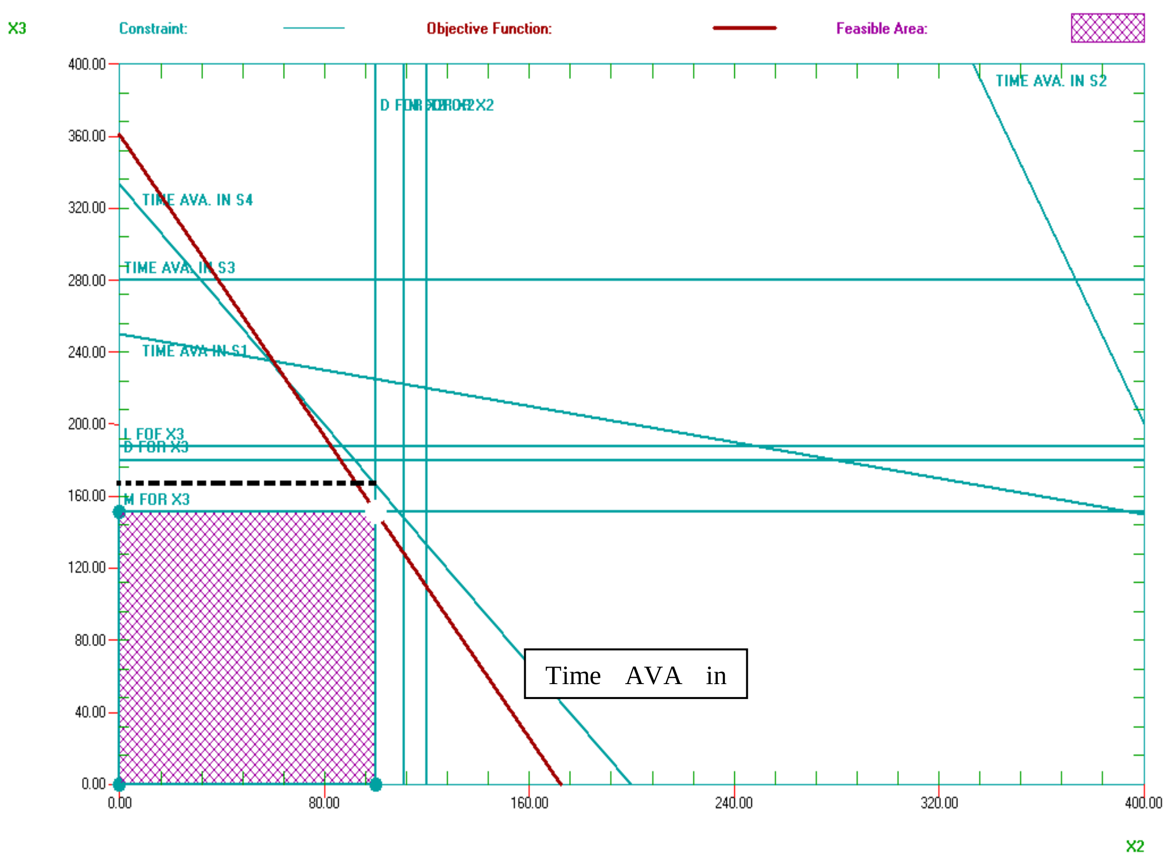
Appendix 2/C: LP Approach to OPM

The mathematical formation of the modified case study

Maximize the following objective function: 80X1 +107X2 +51X3.
Subject to the following constraints:
90X1<=20000 (Capacity Constraints for direct materials for product X1).
45X2<=5000 (Capacity Constraints for direct materials for product X2).
185X3<=28000 (Capacity Constraints for direct materials for product X3).
40X1<=14000 (Capacity Constraints for direct Labour for product X1).
50X2<=6000 (Capacity Constraints for direct Labour for product X2).
80X3<=15000 (Capacity Constraints for direct Labour for product X3).
20X1+5X2+20X3< 9000(Capacity Constraints for process times in station 1).
10X1+15X2+5X3<=9000(Capacity Constraints for process times in station 2).
10X1+0X2+25X3<=9000(Capacity Constraints for process times in station 3).
20X1+25X2+15X3<=9000(Capacity Constraints for process times in station 4).
X1<=200, X2<=100, X3<=180 (Demand constraints).
X1, X2, X3 >=0 (Non-Negativity condition).

Table (2/C/1) the Income statement

Decision Variable	Solution Value	Contribution Margin	Total
X1	200	80	16,000.00
X2	100	107	10,700.00
X3	151.35	51	7,718.9
CM (The Objective Function Max.)			34,418.9
Fixed costs			8,000 -
Total profits			26,418,9



LP Graphical Solution to the Modified Case Study (X3 vs. X2)